

Friedel Hartmann Casimir Katz

---

# Structural Analysis with Finite Elements

With 408 Figures and 26 Tables

# Contents

<b>1</b>	<b>What are finite elements?</b>	<b>1</b>
1.1	Introduction	1
1.2	Key points of the FE method	1
1.3	Potential energy	6
1.4	Projection	8
1.5	The error of an FE solution	13
1.6	A beautiful idea that does not work	15
1.7	Set theory	16
1.8	Principle of virtual displacements	23
1.9	Taut rope	29
1.10	Least squares	33
1.11	Distance inside = distance outside	37
1.12	Scalar product and weak solution	40
1.13	Equivalent nodal forces	42
1.14	Concentrated forces	44
1.15	Green's functions	51
1.16	Practical consequences	55
1.17	Why finite element results are wrong	57
1.18	Proof	64
1.19	Influence functions	69
1.20	Accuracy	80
1.21	Why resultant stresses are more accurate	86
1.22	Why stresses at midpoints are more accurate	88
1.23	Why stresses jump	99
1.24	Why finite element support reactions are relatively accurate	99
1.25	Gauss points	104
1.26	The Dirac energy	110
1.27	How to predict changes	113
1.28	The influence of a single element	126
1.29	Retrofitting structures	130
1.30	Local errors and pollution	136
1.31	Adaptive methods	147
1.32	St. Venant's principle	172
1.33	Singularities	175
1.34	Actio = reactio?	177

1.35	The output .....	181
1.36	Support conditions .....	183
1.37	Equilibrium .....	184
1.38	Temperature changes and displacement of supports .....	187
1.39	Stability problems .....	193
1.40	Interpolation .....	197
1.41	Polynomials .....	199
1.42	Infinite energy .....	208
1.43	Conforming and nonconforming shape functions .....	209
1.44	Partition of unity .....	211
1.45	Generalized finite element methods .....	213
1.46	Elements .....	220
1.47	Stiffness matrices .....	221
1.48	Coupling degrees of freedom .....	224
1.49	Numerical details .....	226
1.50	Warning .....	235
<b>2</b>	<b>What are boundary elements?</b> .....	239
2.1	Influence functions or Betti's theorem .....	240
2.2	Structural analysis with boundary elements .....	247
2.3	Comparison finite elements—boundary elements .....	262
<b>3</b>	<b>Frames</b> .....	269
3.1	Introduction .....	269
3.2	The FE approach .....	270
3.3	Finite elements and the slope deflection method .....	289
3.4	Stiffness matrices .....	292
3.5	Approximations for stiffness matrices .....	298
3.6	Cables .....	305
3.7	Hierarchical elements .....	309
3.8	Sensitivity analysis .....	313
<b>4</b>	<b>Plane problems</b> .....	327
4.1	Simple example .....	327
4.2	Strains and stresses .....	334
4.3	Shape functions .....	337
4.4	Plane elements .....	338
4.5	The patch test .....	344
4.6	Volume forces .....	346
4.7	Supports .....	347
4.8	Nodal stresses and element stresses .....	357
4.9	Truss and frame models .....	363
4.10	Two-bay wall .....	365
4.11	Multistory shear wall .....	365
4.12	Shear wall with suspended load .....	370

4.13	Shear wall and horizontal load . . . . .	375
4.14	Equilibrium of resultant forces . . . . .	378
4.15	Adaptive mesh refinement . . . . .	383
4.16	Plane problems in soil mechanics . . . . .	386
4.17	Incompressible material . . . . .	393
4.18	Mixed methods . . . . .	393
4.19	Influence functions for mixed formulations . . . . .	399
4.20	Error analysis . . . . .	401
4.21	Nonlinear problems . . . . .	401
<b>5</b>	<b>Slabs . . . . .</b>	<b>415</b>
5.1	Kirchhoff plates . . . . .	416
5.2	The displacement model . . . . .	421
5.3	Elements . . . . .	422
5.4	Hybrid elements . . . . .	425
5.5	Singularities of a Kirchhoff plate . . . . .	429
5.6	Reissner–Mindlin plates . . . . .	431
5.7	Singularities of a Reissner–Mindlin plate . . . . .	436
5.8	Reissner–Mindlin elements . . . . .	439
5.9	Supports . . . . .	441
5.10	Columns . . . . .	443
5.11	Shear forces . . . . .	451
5.12	Variable thickness . . . . .	452
5.13	Beam models . . . . .	459
5.14	Wheel loads . . . . .	460
5.15	Circular slabs . . . . .	461
5.16	T beams . . . . .	462
5.17	Foundation slabs . . . . .	469
5.18	Direct design method . . . . .	476
5.19	Point supports . . . . .	477
5.20	Study . . . . .	480
5.21	Sensitivity analysis . . . . .	480
<b>6</b>	<b>Shells . . . . .</b>	<b>485</b>
6.1	Shell equations . . . . .	485
6.2	Shells of revolution . . . . .	488
6.3	Volume elements and degenerate shell elements . . . . .	490
6.4	Circular arches . . . . .	491
6.5	Flat elements . . . . .	493
6.6	Membranes . . . . .	498

XII      Contents

<b>7   Theoretical details . . . . .</b>	<b>503</b>
7.1   Scalar product . . . . .	503
7.2   Green's identities . . . . .	508
7.3   Green's functions . . . . .	516
7.4   Generalized Green's functions . . . . .	519
7.5   Nonlinear problems . . . . .	526
7.6   The derivation of influence functions . . . . .	529
7.7   Weak form of influence functions . . . . .	535
7.8   Influence functions for other quantities . . . . .	539
7.9   Shifted Green's functions . . . . .	541
7.10   The dual space . . . . .	552
7.11   Some concepts of error analysis . . . . .	560
7.12   Important equations and inequalities . . . . .	568
<b>References . . . . .</b>	<b>579</b>
<b>Index . . . . .</b>	<b>593</b>